Quick Reference 8

Truth Functional Operators (Ch. 3)

φ	ψ	$\neg\varphi$	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \rightarrow \psi$	$\varphi \leftrightarrow \psi$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Deduction Rules for TFL (Ch. 4)

Conjunction Introduction

m	φ	
n	ψ	
	$\varphi \wedge \psi$	$\wedge$I m, n

Conjunction Elimination

m	$\varphi \wedge \psi$	
	φ	$\wedge$E m

m	$\varphi \wedge \psi$	
	ψ	$\wedge$E m

Conditional Introduction

i	$\quad\varphi$	Assumption
j	$\quad\psi$	
	$\varphi \rightarrow \psi$	$\rightarrow$I i–j

Conditional Elimination

m	$\varphi \rightarrow \psi$	
n	φ	
	ψ	$\rightarrow$E m, n

Biconditional Introduction

i	$\quad\varphi$	Assumption
j	$\quad\psi$	
k	$\quad\psi$	Assumption
l	$\quad\varphi$	
	$\varphi \leftrightarrow \psi$	$\leftrightarrow$I i–j, k–l

Biconditional Elimination

m	$\varphi \leftrightarrow \psi$	
n	φ	
	ψ	$\leftrightarrow$E m, n

m	$\varphi \leftrightarrow \psi$	
n	ψ	
	φ	$\leftrightarrow$E m, n

Negation Introduction

m	| φ	Assumption
n	| $\bot$	
	$\neg\varphi$	$\neg$I m–n

Negation Elimination

m	φ	
n	$\neg\varphi$	
	$\bot$	$\neg$E m, n

Indirect Proof

m	| $\neg\varphi$	Assumption
n	| $\bot$	
	φ	IP m–n

Disjunction Introduction

m	φ	
	$\varphi \vee \psi$	$\vee$I m

m	φ	
	$\psi \vee \varphi$	$\vee$I m

Disjunction Elimination

m	$\varphi \vee \psi$	
i	| φ	Assumption
j	| χ	
k	| ψ	Assumption
l	| χ	
	χ	$\vee$E m, i–j, k–l

Derived Rules for TFL (§4.11)

Sequent	Derived Rule
$\varphi \to \psi, \neg\psi \vdash \neg\varphi$	MT
$\varphi \vee \psi, \neg\psi \vdash \varphi$	DS
$\varphi \vee \psi, \neg\varphi \vdash \psi$	DS
$\varphi \vdash \psi \to \varphi$	PMI
$\neg\varphi \vdash \varphi \to \psi$	PMI
$\varphi \to \psi \dashv\vdash \neg\varphi \vee \psi$	Imp
$\neg(\varphi \to \psi) \dashv\vdash \varphi \wedge \neg\psi$	NegImp
$\neg(\varphi \wedge \psi) \dashv\vdash \neg\varphi \vee \neg\psi$	DeM
$\neg(\varphi \vee \psi) \dashv\vdash \neg\varphi \wedge \neg\psi$	DeM
$\varphi \dashv\vdash \neg\neg\varphi$	DN
$(\varphi \# \psi) \dashv\vdash (\neg\neg\varphi \# \neg\neg\psi) \dashv\vdash (\neg\neg\varphi \# \psi) \dashv\vdash (\varphi \# \neg\neg\psi)$	SDN
$\neg(\varphi \# \psi) \dashv\vdash \neg(\neg\neg\varphi \# \neg\neg\psi) \dashv\vdash \neg(\neg\neg\varphi \# \psi) \dashv\vdash \neg(\varphi \# \neg\neg\psi)$	SDN
$\varphi @ \psi \vdash \psi @ \varphi$	Com
$\bot \vdash \varphi$	EX
$\vdash \varphi \vee \neg\varphi$	LEM

Deduction Rules for FOL (Ch. 6)

Universal Elimination

$$\begin{array}{l|ll} m & \forall \mathcal{v}\,\varphi(\ldots\mathcal{v}\ldots) & \\ & \varphi(\ldots\mathcal{c}\ldots) & \forall\text{E } m \end{array}$$

Universal Introduction

$$\begin{array}{l|lll} m & \quad \mathcal{c} & & \text{Flag} \\ n & \quad \varphi(\ldots\mathcal{c}\ldots) & & \\ & \forall \mathcal{v}\,\varphi(\ldots\mathcal{v}\ldots) & & \forall\text{I } m\text{–}n \end{array}$$

The Flag-ed name $\mathcal{c}$ may not occur outside the subproof.

Existential Introduction

$$\begin{array}{l|ll} m & \varphi(\ldots\mathcal{c}\ldots) & \\ & \exists \mathcal{v}\,\varphi(\ldots\mathcal{v}\ldots) & \exists\text{I } m \end{array}$$

Existential Elimination

$$\begin{array}{l|ll} m & \exists \mathcal{v}\,\varphi(\ldots\mathcal{v}\ldots) & \\ i & \quad \varphi(\ldots\mathcal{c}\ldots) & \text{Assumption (flag } \mathcal{c}) \\ j & \quad \psi & \\ & \psi & \exists\text{E } m, i\text{–}j \end{array}$$

The Flag-ed name $\mathcal{c}$ may not occur outside the subproof.

Identity Elimination

$$\begin{array}{l|ll} m & \mathcal{a} = \mathcal{b} & \\ n & \varphi(\ldots\mathcal{a}\ldots\mathcal{a}\ldots) & \\ & \varphi(\ldots\mathcal{b}\ldots\mathcal{a}\ldots) & =\text{E } m, n \end{array}$$

$$\begin{array}{l|ll} m & \mathcal{a} = \mathcal{b} & \\ n & \varphi(\ldots\mathcal{b}\ldots\mathcal{b}\ldots) & \\ & \varphi(\ldots\mathcal{a}\ldots\mathcal{b}\ldots) & =\text{E } m, n \end{array}$$

Identity Introduction

$$\begin{array}{|ll} \mathcal{c} = \mathcal{c} & =\text{I} \end{array}$$