

Problem: $(P \rightarrow Q) \vdash (P \rightarrow (A \rightarrow Q))$

1	$(P \rightarrow Q)$	Premise
2	P	Assumption
3	A	Assumption
4	Q	\rightarrow E 1,2
5	$(A \rightarrow Q)$	\rightarrow I 3-4
6	$(P \rightarrow (A \rightarrow Q))$	\rightarrow I 2-5

Problem: $\vdash \neg(P \wedge \neg P)$

1	$(P \wedge \neg P)$	Assumption
2	P	\wedge E 1
3	$\neg P$	\wedge E 1
4	\perp	\neg E 2,3
5	$\neg(P \wedge \neg P)$	\neg I 1-4

Problem: $(A \vee B) \vdash \neg(\neg A \wedge \neg B)$

1	$(A \vee B)$	Premise
2	$(\neg A \wedge \neg B)$	Assumption
3	A	Assumption
4	$\neg A$	\wedge E 2
5	\perp	\neg E 3,4
6	B	Assumption
7	$\neg B$	\wedge E 2
8	\perp	\neg E 6,7
9	\perp	\vee E 1,3-5,6-8
10	$\neg(\neg A \wedge \neg B)$	\neg I 2-9

Problem: $(A \vee \exists xFx) \vdash \exists x(A \vee Fx)$

1	$(A \vee \exists xFx)$	Premise
2	A	Assumption
3	$(A \vee Fa)$	$\vee I$ 2
4	$\exists x(A \vee Fx)$	$\exists I$ 3
5	$\exists xFx$	Assumption
6	Fa	Assumption
7	$(A \vee Fa)$	$\vee I$ 6
8	$\exists x(A \vee Fx)$	$\exists I$ 7
9	$\exists x(A \vee Fx)$	$\exists E$ 5,6-8
10	$\exists x(A \vee Fx)$	$\vee E$ 1,2-4,5-9

Problem: $\vdash \forall x\forall y((Fx \wedge \neg Fy) \rightarrow \neg x = y)$

1	a	Flag
2	b	Flag
3	$(Fa \wedge \neg Fb)$	Assumption
4	$a = b$	Assumption
5	Fa	$\wedge E$ 3
6	$\neg Fb$	$\wedge E$ 3
7	Fb	$=E$ 4,5
8	\perp	$\neg E$ 6,7
9	$\neg a = b$	$\neg I$ 4-8
10	$((Fa \wedge \neg Fb) \rightarrow \neg a = b)$	$\rightarrow I$ 3-9
11	$\forall y((Fa \wedge \neg Fy) \rightarrow \neg a = y)$	$\forall I$ 2-10
12	$\forall x\forall y((Fx \wedge \neg Fy) \rightarrow \neg x = y)$	$\forall I$ 1-11